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Granger Leading and Lagging Relationships Across Different Stages of the Residential Life Cycle: A Case Study of Existing Housing in Taichung City

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Abstract
When the economy is booming, investment demand is strong, housing price fluctuations increase, and housing price fluctuations attract attention. Most studies focus on the impact of external factors on housing price fluctuations. There are few studies on how the time factor of house age affects housing price fluctuations, which creates research motivation. This study aims to explore the relationship between short-term error correction and long-term equilibrium of house prices in different stages. This study will use methods such as VAR vector autoregression model, Johansen co-integration test, Granger causality test, error correction model (vector error correction model (VECM) for short) impact response analysis and variation decomposition. We found that there is a complementary relationship between pre-sale houses, new houses and old houses in the long-term equilibrium relationship between house prices, and that there is a substitution relationship between new and second-hand houses. The short-term error correction lead and lag is a one-way cyclic relationship of pre-sold houses → old houses → new mid-century houses → new houses → pre-sold houses, while mid-century houses lead pre-sold houses and new houses. There is a significant difference in the impact response at different stages. The explanation of the variation of ancient houses and old houses does not reach 50%, while the explanations of the other stages exceed 60%.

Keywords: cointegrating vector, Taichung City, variance, VECM

Introduction

When the economy is booming, investment demand is intense, and housing price fluctuations increase. This fluctuation is mostly affected by external overall economic factors. Therefore, many studies attempt to find factors that influence and explain housing price fluctuations. For example, Fan, et al. (2022) revealed By exploring the relationship between housing price fluctuations and economic policy uncertainty in G7 countries; Wang and Hartzel (2020) Hong Kong's housing fluctuations are mainly affected by exchange rates. Dufitinema (2020) tested whether Finland's housing prices have financial characteristics with assets such as stocks, and tried to explore housing price fluctuations with financial attributes; these are all studies of the impact of external factors on housing price fluctuations, and rarely use the housing price itself to explore the fluctuations in house prices. Therefore, there is research motivation to explore the fluctuation of residential housing prices themselves during economic prosperity.

The age of a house is the only factor with time that can be used to study housing price fluctuations. Other factors other than the age of a house will hardly change over time (Liang Renxu, 2015). They are often included as explanatory variables in house price research and analysis; this study uses the life cycle of a house based on The house price is divided into several stages. Its purpose is to explore the explanation of house price on its own house price at different stages, observe the leading and lagging relationship of short-term housing price error process correction and the long-term equilibrium relationship of housing prices, and predict short-term and long-term housing prices. Fluctuation correction process.

The life cycle of a house begins with the issuance of a building license. Houses that can be built and sold after the issuance are called pre-sale houses. The new house has experienced physical, functional and functional depreciation over time, resulting in it not being suitable for current living conditions. standards, and the safety and quality of residences are considered for reconstruction. The process from the issuance of a building license to demolition and reconstruction is called the "residential life cycle"; the attributes and use values of each stage of the residential life cycle are different. For example, pre-sale houses have futures trading Properties, due to the risk of backward information and uncertain housing delivery, the price should be lower than that of newly-established houses, the construction and maintenance costs of new houses are low, and the living functions are in line with the needs of the time. The second-hand houses are the main ones with the largest number of houses. It is the mainstream of residential transactions. Compared with buying old houses, the maintenance cost is high and the quality is poor compared to other houses. However, it has the value of reconstruction, which implies that the value of each stage of the housing cycle is different. The fluctuation of housing prices is short-term and long-term. The error correction process is worth observing.

Each stage of the residential life cycle in this study is divided into pre-sale houses, new houses (houses less than 3 years old), new and second-hand houses (houses more than 3 to less than 15 years old), and second-hand houses (houses more than 15 to 30 years old) according to the age of the house.) and old houses (more than 30 years old). This study will use VAR vector autoregression model, Johansen co-integration test to verify the long-term equilibrium of housing prices, Granger causality test, error correction model (vector error correction model (VECM) shock response analysis and variation decomposition) and other methods to verify the Taichung City Housing prices in each stage of the housing life cycle are corrected for short-term errors in their leading and lagging relationships. We find that there is a complementary relationship between pre-sold homes, new homes and old homes, with a correction coefficient of -0.097 for the long-term relationship and short-term error correction. The housing prices of pre-sold houses, new houses and old houses form a one-way cyclical relationship of new houses → pre-sold houses → old houses → new second-hand houses, and second-hand houses lead the pre-sale houses and new houses. There are significant differences in the impact responses at different stages. The explanation degree of the variation in ancient houses and old houses does not reach 50%, while the explanation degree in other stages exceeds 60%.

Literature Review

1. The Impact of Housing Age on Market Value

The residential lifecycle reflects the social background, housinghold income, population structure, regulations for residential safety facilities, and construction techniques at the time of the housing's existence, providing different mainstream spaces and housing functions. As economic capacity increases, quality demands become higher, and the management and maintenance of housing at various stages cannot meet current usage needs. Therefore, each residential cycle represents different residential space needs, leading to variations in price ranges. For example, with Taiwan's rapidly increasing elderly population, elevator facilities have become a basic feature of newly built housing, which was not common 20 years ago, highlighting the inability of older housing to meet current demands. Generally, the residential lifecycle can be divided into pre-sale housing, new housing, and second-hand housing. In Japan, the Ministry of Land, Infrastructure, Transport, and Tourism (MLIT) conducted the "Comprehensive Survey of Housing and Living Conditions" from 2003 to 2018, showing that both owner-occupiers and tenants prefer new housing over second-hand housing. Masatomo Suzuki et al. (2022) found that structural value depreciates by up to 50% within the first 10 years after construction, with depreciation continuing for about 30 years. This means that during the early stages of a building's life (or between new and relatively new second-hand housing), the demand for "novelty" is significant, with the rate of structural value loss far exceeding the measured speed of decline in the latter stages of a building's life. Therefore, the entirely new functions of new housing better meet modern living needs, leading to a strong "novelty" demand and a surge in pre-sale housing.

What are the definitions of pre-sale housing, new housing, and second-hand housing? Currently, the Real Estate Brokerage Management Act Article 4 in Taiwan defines pre-sale housing as "properties that have obtained a construction permit but have not yet been completed and are traded as future completed buildings." Other definitions are not specified, but information from real estate websites such as Housingfun News (2022-01-15) and H&B Business group. (https://www.hbhousing.com.tw/News/Detail.aspx?Num=4965) provide market-recognized definitions. The government informally recognizes second-hand housing as those that have been transferred at least once or have been completed and have a usage license for more than three years. The "Urban Dangerous and Dilapidated Buildings Accelerated Reconstruction Regulations" defines housing over 30 years old as old housing. The website of H&B Business group defines new housing as just completed, unsold housing with an age of less than two years and no previous occupants, including remaining housing after pre-sales. Additionally, from the perspective of value retention, subsequent maintenance costs, and community management, housing aged 5-8 years are considered the golden age, while housing over 15 years old are considered cost-effective choices due to lower public facilities costs. This is because, starting in 2007, regulations required buildings over eight floors to add two escape staircases, increasing public facilities costs in new projects, with the dividing point around 15 years. Furthermore, when analyzing mortgage loan conditions, the loan period generally has a negative correlation with housing age. The principle is that for housing aged 25 years, the maximum loan period is 25 years; for housing aged 30 years, the maximum loan period is 20 years; and for housing aged 40 years, the maximum loan period is 10 years. Thus, with the same loan amount, shorter terms result in higher monthly mortgage payments, and higher housing ages have shorter repayment periods, making it more difficult to obtain transaction funding.

2. Literature on Housing Depreciation

Hulten & Wykoff (1981) define depreciation as the reduction in asset value caused by housing age. Clapp & Guncotto

(1998) further explain that depreciation results from increased maintenance costs and decreased usability due to the aging of the property, leading to a decline in value. Liang (2015)The characteristics of real estate parcels, aside from housing age, generally do not change over time; and housing age is almost always included as an explanatory variable in housing price analyses (). In Taipei City, apartment rents decrease as housing age increases, while real estate transaction prices initially decrease and then increase with housing age. This results in a capitalization rate influenced by both factors, showing a phenomenon where the rate initially rises and then falls with housing age. In areas with frequent turnover of old and new buildings, the substantial downward fluctuation in prices of high-aged properties causes the capitalization rate to rise and exhibit low stability. Masatomo Suzuki et al. (2022) found that structural value loss can be as high as 50% in the first 10 years after construction, with depreciation continuing until the property reaches around 30 years old. This indicates that depreciation in the early stages of a building's life (or between new and relatively newer resale properties) is significantly pronounced due to a "novelty effect." Mats Wilhelmsson (2008) found in their study of various real estate markets that average values decline with age, and in some markets, the rate of depreciation deviates significantly from the average level.

3. Literature on Cointegration Application

Chiang et al. (2011) studied the long-term relationship between Taiwan's real estate market and financial variables, finding a long-term cointegration relationship among multiple variables. The long-term error correction function among the variables was also significant. In domestic research on second-hand housing and price relationships, it was found that there is a long-term equilibrium relationship between second-hand housing and foreclosure auction prices, with different correction speeds and lead-lag relationships between the variables. Tsai et al. (2022) used the VAR (Vector Autoregression) model to verify the impact of various supply and demand variables on housing prices. The VAR model is a multivariate time series model that handles the mutual influence of multiple variables and their changes over time. The study found bidirectional causality between the total amount of housing loans by the five major banks and housing prices, between mortgage rates and housing prices, and a positive relationship between unsold housing and housing prices, with mortgage rates having the greatest impact on housing prices among all variables. Lee et al. (2014) pointed out that Taipei and Kaohsiung are long-term leaders in housing prices. Granger causality test results revealed a strong lead-lag interdependence in housing price changes among major urban areas in Taiwan, with pre-sale housing price trends being random in Taiwan's largest urban areas.Hung (2008) studied second-hand housing prices and pre-sale housing prices using the Sinyi housing price index from the third quarter of 1991 to the fourth quarter of 2007 and the Cathay housing price index from the first quarter of 1997 to the fourth quarter of 2007. The conclusion was that stock prices lead second-hand housing prices, and pre-sale housing prices lead stock prices, indicating that the wealth effect driven by rising stock prices helps stimulate the relatively low-priced second-hand housing market but has less utility in the pre-sale housing market. The vector error correction model showed that previous stock prices positively influenced second-hand housing prices and negatively influenced pre-sale housing prices. Therefore, for investors, the stock market and second-hand housing market are complementary, while the stock market and pre-sale housing market are substitutes. Chen et al. (2022) In the case of G7 countries, different causal relationship estimates indicate that over time, stocks and housing assets are the most important forms of assets.

The collected literature in this study can be divided into two types of data:Housing prices and macroeconomic factors such as interest rates, exchange rates, consumer price index, and gross domestic product.Housing prices explained by housing prices themselves, including cross-national cities, domestic cities, and different types of real estate, such as second-hand and foreclosed housing.The commonly used methods to verify research purposes are the Granger causality test and cointegration test, which have their stability. (Laing 2015) Housing age is included as an explanatory variable in almost all housing price analyses. This study divides the residential cycle into several stages based on housing age to explore its ability to explain housing price variables. Compared to other economic variables, housing age has a greater explanatory advantage. The study will establish the most appropriate model to verify the long-term and short-term trends of housing prices and the interrelationships between housing prices at different stages.

Researc Methods

1. Research Hypotheses

Hua et al. (2001) utilized a time series analysis to detect the long-term equilibrium between pre-sale prices and existing prices and their adjustment speeds. Their findings indicated that both pre-sale and existing prices tend to reach long-term equilibrium, but pre-sale prices adjust more quickly, suggesting that the pre-sale system enhances market efficiency. Hung (2008) utilized the following methods: 1. Causality tests. 2. Cointegration tests 3. Vector Error Correction Model (VECM). The results showed that stock prices lead the prices of second-hand housing, and pre-sale housing prices lead stock prices. Stock price increases create a wealth effect that stimulates the second-hand housing market more than the pre-sale housing market. The VECM revealed that previous stock prices positively impact second-hand housing prices but negatively impact pre-sale housing prices. The stock market and the second-hand housing market are complementary, while the stock market and the pre-sale housing market are substitutes. Jiang, et al. (2011) used Hansen and Seo's threshold error correction model. Their results indicated that the prices in these two markets lead and lag each other. There is a long-term equilibrium relationship between the second-hand housing price index and the auctioned housing price index, and the explanatory power of the threshold error correction model is better than that of the traditional error correction model. Lee, et al. (2014) applied 1. Cointegration analysis 2. Granger causality tests. Their results showed that Taipei City did not deviate from the long-term price trend, and there is a strong leading-lag interdependence in housing price changes among major cities in Taiwan..Chen. et al. (2022) applied 1. Granger causality model 2. Causality tests. Their findings showed no causality with the UK, unidirectional causality with Canada, Italy, Japan, and the United States, and bidirectional causality with France and Germany (before the euro introduction). Tsai. et

al. (2022) used a VAR (Vector Autoregression) model to examine the impact of various supply and demand variables on housing prices. The VAR model is a multivariate time series model that simultaneously handles the interactions between multiple variables and their changes over time. The study found that there is a bidirectional causal relationship between the total mortgage amount handled by the five major banks and housing prices, as well as between mortgage interest rates and housing prices. Unsold properties and housing prices exhibit synchronous changes. Among all the variables, mortgage interest rates have the highest impact on housing prices.

Based on the above, this study establishes the following hypotheses:

Hypothesis 1. Verify whether there is a co-integration relationship between the long-term price trends of each stage of the housing life cycle.

Hypothesis 2. Verify whether there is a leading or lagging relationship in the short-term error correction of price fluctuations in each stage of the housing life cycle.

Hypothesis 3. Whether the self-explanation ability of house prices at each stage reaches 50% explanation ability.

2. Research Variables and Process.

(1) .Research Variables

This study explores the relationship between the variables of pre-sale housing$(y_{1,t})$, new second-hand housing$(y_{2,t})$ second-hand housing$(y_{3,t})$ new housing$(y_{4,t})$ and old housing$(y_{5,t})$. The explanation of related research variables is as follows. Based on the literature review, the annual cycle variables of residential housing are shown in Table 1.

Table 1: Duration of Each Stage in the Housing Cycle

Housing Types	Pre-sale housing $(y_{1,t})$	New housing $(y_{2,t})$	New-second housing $(y_{3,t})$	Second housing $(y_{4,t})$	Old housing $(y_{5,t})$
Stage Duration (year)	–	$0 \sim \leq 3$	beyond $3 \sim \leq 15$	beyond $15 \sim \leq 30$	beyond 30year

1. Pre-sale housing variable: refers to properties that have obtained construction permits but have not yet been completed and are traded as future completed buildings.2. New housing: originally referred to housing within two years and not yet occupied, adjusted to housing within three years.3.New second-hand housing: due to the increase in public facilities to double escape stairs since 2007, housing within 15 years have a similar proportion of public facilities.4. Second-hand housing: refers to housing over 15 years and within 30 year.5. Old second-hand housing: housing over 30 years old.

(2). Transaction Data Sorting

The data period is from August 2012 to June 2023, with a total of 489,448 real transaction records of residential housing in Taichung City. For data consistency, after excluding missing and outlier data, there are 242,957 valid records of exist housing and76,648 valid records of per-salet housing . As shown in Table 2 and Table3,

Table 2: Data Filtering Process

Step	Number of Exclusions	Valid Data (Records)
Population Data	-	489,448
Housing and Land Integration	142,999	346,449
Residential Products	100,453	245,996
Removal of Missing Values	3,015	242981
Removal of Outliers (Using 3 Standard Deviations)	24	242,957
total	246,491	242,957

Table 3: Data Filtering Process for Pre-Sale housing

Step	Number of Exclusions	Valid Data (Records)
Population Data	-	79,098
Housing and Land Integration	954	78144
Residential Products	564	77580
Removal of Missing Values	932	76648
total	2,450	76,648

there are 242,957valid transaction records after screening.Residential housing types are divided into four categories:1. High-rise apartments (below 10 stories with elevators).2. High-rise buildings (of at least 11 stories with elevators).3. townhome (with a unique housing number).4. Apartments (of at most five stories without elevators)s. They are classified by building age into pre-sale housing, new housing, new second-hand housing, second-hand housing and old housing. The relationship between housing products and data is shown in Table 4: New housing have the highest transaction ratio in high-rise buildings at 62.63%, followed by row housing at 30.05%. New second-hand housing are similar to new housing, with high-rise buildings accounting for the highest ratio at 63.38%, followed by townhome at 26.48%. Second-hand housing have the highest ratio of high-rise buildings at 46.74%, followed by apartment buildings at 29.83%. Old housing have the highest ratio of townhome at 56.81%, followed by Apartments at 28.21%. It is observed that in recent years, with the advancement of construction technology, housing uses land more intensively, with high-

rise buildings being the mainstream of transactions. Therefore, as shown in Table 4, new housing and new second-hand housing have the highest ratio of high-rise buildings. For old housing, townhome account for the highest proportion, followed by Apartments, which have the most potential for reconstruction. There are a total of 131 monthly transaction price periods for each cycle and pre-sale housing, as shown in Table 5.

Table 4: Distribution of Housing Cycle Stages and Housing Type Transaction Volumes

Housing age Housing Types	New housing≤3	New-second housing <3~≤15	Second housing15~≤30	Old housing beyond30year
High-rise apartments	4,730	3,982	26,600	3,794
%	7.07%	8.96%	29.83%	8.95%
High-rise buildings	41,943	28,159	41,689	2,555
%	62.63%	63.38%	46.74%	6.03%
Townhouse	20,126	11,765	16,195	24,071
%	30.05%	26.48%	18.16%	56.81%
Apartments	166	526	4,702	11,954
%	0.25%	1.18%	5.27%	28.21%
合計	66,965	44,432	89,186	42,374
%	100%	100%	100%	100%

period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
monthly	2012M8	2012M9	2012M10	2012M11	2012M12	2013M1	2013M02	2013M3	2013M4	2013M5	2013M6	2013M7	2013M8	2013M9	2013M10	2013M11
New housing	2.908	2.918	2.895	3.173	2.96	2.925	2.966	2.97	2.962	3.061	3.029	2.975	2.918	3.065	2.972	3.059
New-second housing	2.738	3.017	2.804	2.852	2.937	2.833	2.862	2.859	3.028	2.948	2.938	2.924	2.927	2.894	2.965	3.05
second housing	2.666	2.796	2.482	2.496	2.486	2.501	2.528	2.501	2.679	2.712	2.566	2.621	2.601	2.704	2.679	2.694
Old housing	2.817	2.779	2.841	2.83	2.743	2.759	3.01	2.837	2.872	2.968	3.062	2.845	2.855	2.976	2.95	2.648
Pre-sale housing	2.694	2.776	2.838	2.917	2.861	2.906	2.955	2.904	2.810	2.974	2.843	3.059	2.952	2.860	2.847	2.921
period	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
monthly	2013M12	2014M1	2014M2	2014M3	2014M4	2014M5	2014M6	2014M7	2014M8	2014M9	2014M10	2014M11	2014M12	2015M1	2015M2	2015M3
New housing	2.92	3.008	2.988	3.065	3.02	3.108	2.99	3.028	3.024	2.844	2.976	3.05	3.035	3.175	3.01	3.053
New-second housing	3.051	2.972	2.685	3.043	3.127	2.989	2.954	2.999	3.023	2.956	3.107	3.103	3.19	3.026	2.978	2.954
second housing	2.66	2.649	2.691	2.651	2.816	2.687	2.664	2.673	2.754	2.788	2.657	2.823	2.664	2.745	2.74	2.75
Old housing	2.898	3.065	2.984	2.939	3.11	2.941	2.967	2.916	3.057	3.131	2.693	3.045	3.16	2.899	2.973	3.097
Pre-sale housing	2.828	2.859	2.824	2.997	3.157	3.220	3.278	3.200	3.112	3.097	2.964	3.370	3.352	3.294	3.211	3.357
period	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
monthly	2015M4	2015M5	2015M6	2015M7	2015M8	2015M9	2015M10	2015M11	2015M12	2016M1	2016M2	2016M3	2016M4	2016M5	2016M6	2016M7
New housing	3.014	3.098	3.073	3.34	3.111	3.096	3.063	3.081	3.099	3.083	3.102	3.121	3.184	3.085	3.176	3.071
New-second housing	3.094	3.057	3.031	3.177	3.032	3.019	3.138	3.057	2.97	3.048	3.063	3.077	3.109	3.08	3.159	3.079
second housing	2.719	2.767	2.704	3.597	2.675	2.715	2.757	2.736	2.714	2.714	2.679	2.643	2.727	2.733	2.824	2.661
Old housing	3.026	2.96	2.893	3.046	2.833	2.994	2.969	2.848	2.71	2.905	2.872	2.837	2.995	2.979	3.162	2.776
Pre-sale housing	3.342	3.226	3.328	3.303	3.515	3.450	3.411	3.359	3.549	3.406	3.330	3.399	3.356	3.238	3.169	3.162

period	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
monthly	2016M8	2016M9	2016M10	2016M11	2016M12	2017M1	2017M2	2017M3	2017M4	2017M5	2017M6	2017M7	2017M8	2017M9	2017M10	2017M11
New housing	3.043	3.087	3.05	3.059	3.067	2.999	3.154	3.47	3.027	3.117	3.135	3.206	3.085	3.075	3.091	3.107
New-second housing	3.092	3.107	3.107	3.073	3.037	3.144	3.086	3.125	2.951	3.087	3.059	3.073	3.118	3.057	3.086	3.114
second housing	2.746	2.797	2.773	2.748	2.723	3.059	2.715	2.769	2.612	2.735	2.78	2.729	2.787	2.695	2.716	2.737
Old housing	3.038	2.983	2.964	2.928	2.891	2.96	3.025	3.055	2.451	3.004	3.008	3.069	3.015	3.041	3.001	2.959
Pre-sale housing	3.110	3.137	3.191	3.263	3.116	3.183	3.464	3.362	3.293	3.196	3.091	3.188	3.319	3.334	3.325	3.392
period	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
monthly	2017M12	2018M1	2018M2	2018M3	2018M4	2018M5	2018M6	2018M7	2018M8	2018M9	2018M10	2018M11	2018M12	2019M1	2019M2	2019M3
New housing	3.091	2.957	3.173	3.121	3.172	3.103	3.031	3.173	3.158	3.195	3.15	3.066	3.105	3.122	3.098	3.123
New-second	3.044	2.929	3.121	3.104	3.105	3.088	3.058	3.159	3.102	3.135	3.135	3.128	3.123	3.088	3.153	3.118

housing																
second housing	2.737	2.602	2.781	2.776	2.74	2.724	2.65	2.765	2.756	2.788	2.77	2.744	2.777	2.738	2.806	2.781
Old housing	3.08	2.659	3.173	3.093	3.007	2.938	2.924	3.201	2.962	2.991	3.05	3.079	3.027	3.332	3.341	3.154
Pre-sale housing	3.392	3.386	3.334	3.316	3.325	3.232	3.193	3.148	3.190	3.233	3.132	3.142	3.214	3.138	3.141	3.116
period	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
monthly	2019M4	2019M5	2019M6	2019M7	2019M8	2019M9	2019M10	2019M11	2019M12	2020M1	2020M2	2020M3	2020M4	2020M5	2020M6	2020M7
New housing	3.134	3.177	3.149	3.195	3.223	3.189	3.217	3.212	3.21	3.268	3.183	3.149	3.203	3.231	3.135	3.213
New-second housing	3.162	3.139	3.152	3.153	3.153	3.161	3.16	3.159	3.169	3.179	3.157	3.167	3.146	3.148	3.292	3.204
second housing	2.815	2.831	2.836	2.849	2.862	2.936	2.807	2.81	2.849	2.842	2.862	2.866	2.863	2.887	2.864	2.968

period	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112
monthly	2020M8	2020M9	2020M10	2020M11	2020M12	2021M1	2021M2	2021M3	2021M4	2021M5	2021M6	2021M7	2021M8	2021M9	2021M10	2021M11
New housing	3.213	3.294	3.223	3.21	3.217	3.282	3.281	3.297	3.332	3.351	3.313	3.323	3.335	3.363	3.33	3.333
New-second housing	3.218	3.235	3.288	3.27	3.254	3.247	3.271	3.268	3.341	3.322	3.309	3.273	3.309	3.342	3.348	3.346
second housing	2.924	2.942	2.982	2.983	3.019	3.01	3.075	2.987	3.014	3.216	3.072	3.001	3.032	3.073	3.047	3.056
Old housing	3.226	3.224	3.275	3.166	3.248	3.299	3.227	3.17	3.269	3.227	3.287	3.218	3.179	3.267	3.249	3.319
Pre-sale housing	3.311	3.299	3.330	3.325	3.370	3.572	3.437	3.504	3.424	3.425	3.358	3.495	3.435	3.396	3.428	3.423
period	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128
monthly	2021M12	2022M1	2022M2	2022M3	2022M4	2022M5	2022M6	2022M7	2022M8	2022M9	2022M10	2022M11	2022M12	2023M1	2023M2	2023M3
New housing	3.358	3.431	3.457	3.508	3.497	3.492	3.449	3.469	3.419	3.472	3.407	3.441	3.464	3.5	3.436	3.425
New-second housing	3.38	3.448	3.491	3.432	3.446	3.443	3.426	3.479	3.418	3.501	3.486	3.492	3.492	3.422	3.443	3.475
second housing	3.087	3.092	3.171	3.126	3.139	3.146	3.149	3.202	3.201	3.183	3.237	3.187	3.172	3.191	3.221	3.265
Old housing	3.22	3.312	3.232	3.292	3.281	3.31	3.269	3.41	3.282	3.387	3.347	3.273	3.259	3.276	3.277	3.355
Pre-sale housing	3.457	3.617	3.521	3.606	3.705	3.638	3.687	3.730	3.650	3.664	3.726	3.720	3.653	3.679	3.681	3.695
period	129	130	131													
monthly	2023M4	2023M5	2023M6													
New housing	3.47	3.509	3.505													
New-second housing	3.469	3.483	3.536													
second housing	3.254	3.251	3.229													
Old housing	3.171	3.267	3.257													
Pre-sale housing	3.728	3.701	3.739													

3. Methods of variable analysis

(1) Johansen Cointegration Test

The Johansen cointegration test examines whether there is a cointegration relationship among multiple integrated variables. It can be applied to multiple groups of cointegration vectors. Suppose we haveX_t=[$X_{1t,}X_{2t,}$ $X_{3t,}$…$X_{nt,}$], where each variable is related to the lagged values of the other variables. This can be expressed as:X_t=AX_{t-1}+ε_t By subtracting the lagged variable from both sides of the equation, we get:X_t=(A-I)X_{t-1}+ε_t Rewriting this equation, we have:X_t=ΠX_{t-1}+ε_t . we can determine n eigenvalues, ordered as $\lambda_1 > \lambda_2 > \cdots > \lambda_n$, If these n non-stationary variables are all independent, then rank(Π)=0, and $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$, Therefore, the natural logarithm ln n(1−λ_i)=ln(1)=0 for all i ; When rank(Π)=1,$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = \cdots .. = 0$。 When rank(Π)=2,$\lambda_1 \neq 0, \lambda_2 \neq 0, \lambda_3 = \lambda_4 = \cdots . = \lambda_n = 0$.The cointegration test can be performed using two statistical tests (where T represents the sample size, $\hat{\lambda}_i$ is the estimated i-th eigenvalue. Trace test λ_{trace}(r)=−T$\sum_{i=r+1}^{n} \ln(1-\hat{\lambda}_i)$,The principle of this test is that if no cointegration vectors exist γ =0. If there are$\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$,$\lambda_{trace}$=0 . If there exist γ gammaγ cointegrating vectors , $\lambda_1 \neq 0, \lambda_2 \neq 0, \ldots\ldots$, $\lambda_r \neq 0$,but$\lambda_{r+1} = \lambda_{r+2} = \cdots = \lambda_n = 0$.$\lambda_{trace}$(r)=−T$\sum_{i=r+1}^{n} \ln(1-\hat{\lambda}_i)$ values will approach zero.Maximum Eigenvalue Test (λ_max): $\lambda_{max}(\gamma, \gamma+1)$=−Tln(1−$\hat{\lambda}_{r+1}$)The principle of this test is that if there are no cointegration vectors, $\lambda_1 \neq 0, \lambda_2 \neq 0, \ldots\ldots, \lambda_r \neq 0$,but $\lambda_{r+1} = \lambda_{r+2} = \cdots = \lambda_n = 0$, then forγ+1 eigenvalues, $\lambda_{r+1} = 0$.$\lambda_{max}(\gamma, \gamma+1)$=−Tln(1−$\hat{\lambda}_{r+1}$) will approach zero.(Yang, 2017).

(2). Vector Autoregression Model (VAR)

The direction of short-term price correction should move toward the long-term equilibrium price. When the short-term price deviates more from the equilibrium price, the correction

magnitude becomes larger. Naturally, when the short-term price equals the long-term price, no adjustment is needed. This is known as error correction, which is essentially a concept of the dynamic adjustment mechanism in an economic system. A model that includes an error correction term is called a Vector Error Correction Model (VECM).Assuming the three variables are$\{x_t\}$、$\{y\}$ and $\{z_t\}$,that they are CI(1,1), the long-term equilibrium relationship of the variables can be expressed as:

$y_t = \beta_0 + \beta_1 + \beta_2 z_t + e_t$.Then, the form of the VECM for the three variables is:

$$\triangle y_t = \alpha_1 + \alpha_y(y_{t-1} - \beta_1 x_{t-1} - \beta_2 z_{t-1}) + \sum_{i=1} \alpha_{11}(i) \triangle x_{t-i} + \alpha_{12}(i) \triangle y_{t-i} + \alpha_{13}(i) \triangle z_{t-i} + \varepsilon_{yt}$$

$$\triangle x_t = \alpha_2 + \alpha_x(y_{t-1} - \beta_1 x_{t-1} - \beta_2 z_{t-1}) + \sum_{i=1} \alpha_{21}(i) \triangle x_{t-i} + \alpha_{22}(i) \triangle y_{t-i} + \alpha_{23}(i) \triangle z_{t-i} + \varepsilon_{xt}$$

$$\triangle z_t = \alpha_2 + \alpha_z(y_{t-1} - \beta_1 x_{t-1} - \beta_2 z_{t-1}) + \sum_{i=1} \alpha_{31}(i) \triangle x_{t-i} + \alpha_{32}(i) \triangle y_{t-i} + \alpha_{33}(i) \triangle z_{t-i} + \varepsilon_{xt}$$

In the equations, α_x、α_y and α_z represent the adjustment speed coefficients. After taking their absolute values, they should not be too large. The long-term convergence relationship of $\triangle x_t$、$\triangle y_t$ and $\triangle z_t$ is estimated based on these coefficients. Johansen cointegration tests can be used to determine whether there exists a cointegration relationship. If no cointegration relationship is found, VAR analysis should be conducted instead.

(3).Granger Causality Test.

When past information of variable X helps in predicting variable Y, X is said to Granger-cause Y, and Y is the effect of X. Conversely, if past information of Y helps in predicting X, Y Granger-causes X. Granger (1969) proposed determining whether a causal relationship exists between variables based on the accuracy of predictions made using the variables. This concept of causality is further explored in financial analysis to investigate the "leading" or "lagging" relationships between variables. The process of testing for causality can be illustrated using a bivariate regression model:

$$X_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-1} + \sum_{j=1}^{q} \beta_j Y_{t-j} + \varepsilon_{1t}$$

$$Y_t = \gamma_0 + \sum_{i=1}^{p} \gamma_i X_{t-1} + \sum_{j=1}^{q} \delta_j Y_{t-j} + \varepsilon_{2t}$$

The null hypotheses for the F-statistic joint test are

$$\mathcal{H}_0: \beta_1 = \beta_2 = \cdots = \beta_j = 0 \ ; \ H_0 : \gamma_1 = \gamma_2 = \cdots = \gamma_j = 0$$

The restricted models are expressed as follows:

$$X_t = \sum_{i=1}^{p} \alpha_i X_{t-1} + \mu_t \ ; \ Y_t = \sum_{j=1}^{q} \delta_j Y_{t-j} + \nu_t.$$

Results and Discussion of Empirical Analysis

1. Estimating and Determining Lag Length

(1). Price Data Trends and Nature

From August 2012 to June 2023, the price data trends for new homes, new and second-hand homes, and old homes are illustrated in Figure 1. The trends, ranked from high to low, are as follows: pre-sale homes, new homes, and new second-hand homes, followed by old homes and second-hand homes.



Fig 1: Trend of Transaction Prices across Different Cycles

Observing Figure 2, we analyze the characteristics of non-stationary variables and time trends to select the testing form. In this study, all transaction price variables are converted to natural logarithms. The exogenous variables include new homes, new second-hand homes, second-hand homes, old homes, and pre-sale homes. These variables seem to exhibit unit roots with intercepts but do not show significant time trends. Based on the Johansen test, we may consider not including time trend terms.



Fig 2: Trend of Transaction Prices across Different Cycles in Natural Logarithm

(2). Unit Root Testing of Variables

The null hypothesis H0 is "accepting unit root," and the alternative hypothesis H1 is "rejecting unit root." We test for unit roots using the Augmented Dickey-Fuller (ADF) test statistic, without lags and exogenous variables but including intercepts, to check if the time variables are stationary. For the original home age variable with intercepts, the ADF test results show non-stationarity ,as show in Table 6.

Table 6: Unit Root Test of Raw Data

Variable	Pre-sale housing	New housing	New-second housing	second housing	Old housing
Statistic	-2.55	-0342	0.309	-0.510	-1.627
p-value	0.130	0.914	0.978	0.884	0.466
H_0/H_1	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0
Decision	Non-Stationary	Non-Stationary	Non-Stationary	Non-Stationary	Non-Stationary

Note: $\mathcal{P}^{*} \leq 0.05$; $\mathcal{P}^{**} \leq 0.01$; $\mathcal{P}^{***} \leq 0.000$, H_0 (Null Hypothesis)/H_1 (Alternative Hypothesis).

After first differencing, the ADF test results indicate stationarity, as show in Table 7.

Table 7: Unit Root Test after first-differencing

Variable	Pre-sale housing	New housing	New-second housing	second housing	Old housing
Statistic	-14.164	-11.385	-11.494	-11.293	-13.139
p-value	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}	0.0000^{***}
H_0/H_1	Accept H_1	Accept H_1	Accept H_1	Accept H_1	Accept H_1
Decision	Stationary	Stationary	Stationary	Stationary	Stationary

Note: $\mathcal{P}* \leq 0.05$; $\mathcal{P}** \leq 0.01$; $\mathcal{P}*** \leq 0.000$, H0 (Null Hypothesis)/H1 (Alternative Hypothesis).

(3).Selecting Lag Length

The selection of lag length in the VAR model, as shown in Table 8 , indicates that the Akaike Information Criterion (AIC) suggests a lag length P=3at the 5% significance level. The Schwarz Criterion (SCB) and Hannan-Quinn Criterion (HQC) methods suggest a lag length P=1 at the significance level. The Likelihood Ratio (LR) test indicates that P=11 is significant. The comparison of lag lengths P=1, P=3,P=11 shows that only P=1indicates cointegration. Therefore, this study decides to choose P=1 as the lag length.

Table 8: Determination of Lag Length

Lag	LogL	LR	FPE	AIC	SC	HQ
0	484.7959	NA	2.17E-10	-8.063797	-7.947027	-8.016380
1	663.8658	340.0823	1.63E-11	-10.65321	-9.952587*	-10.36871*
2	687.1480	42.26027	1.68E-11	-10.62434	-9.339868	-10.10275
3	719.3933	55.81953	1.49E-11*	-10.74611	-8.877788	-9.987441
4	740.1430	34.17594	1.62E-11	-10.67467	-8.222504	-9.678924
5	761.1307	32.80434	1.76E-11	-10.60724	-7.571221	-9.374408
6	785.1083	35.46269	1.84E-11	-10.59006	-6.970189	-9.120142
7	814.5339	41.04739	1.77E-11	-10.66443	-6.460718	-8.957438
8	830.8761	21.42341	2.15E-11	-10.51893	-5.731360	-8.574846
9	862.8839	39.27007	2.04E-11	-10.63670	-5.265288	-8.455541
10	893.0056	34.42490	2.04E-11	-10.72278	-4.767519	-8.304539
11	934.6846	44.13069*	1.71E-11	-11.0031*	-4.463989	-8.347775
12	952.3790	17.24826	2.21E-11	-10.88032	-3.757355	-7.987908

Note:* Minimum Value (Optimal Value

2. Cointegration Vector Testing - Presence of Trend Terms

In the Johansen model, we reestimate the model with constraints to test for the presence of intercept terms using the Likelihood Ratio (LR) statistic. The null hypothesis is H_0: $C_{10}= C_{20}= C_{30}= C_{40}=C_{50}=0$, which implies acceptance of the intercept term. If H_0 is rejected, then the intercept term is not accepted, whereC_n represents the n-th intercept term.

This study identifies three cointegration vectors. The maximum likelihood value of the logarithm of the unrestricted model (LU) is better with intercept terms, while the restricted model (LR) without intercept terms is better. The statistic follows a chi-square distribution, as described in Equation (4-3):

$LR=-2(L_R-L_U) \sim X^2(m)$ m=4 .....Eq1

The decision rule for the maximum likelihood ratio test is: If LR> $X^2(m)$, the unrestricted model (L_U)) is preferred, rejecting the null hypothesis of the restricted model. If LR< $X^2(m)$, the restricted model (L_R) is preferred, accepting the null hypothesis. As Table 9, Since LR> $X^2(m)$ at the 5% critical value, this study's error correction model includes intercept terms.

Table 9: Test of restricted formula

No	LR	LU	LR值	X2(m)5%臨界值
1*	678.19	684.21	12.04	X2(1)=3.84
2*	701.12	712.03	21.82	X2(2)=5.99
3*	721.93	732.84	21.82	X2(3)=7.81
4	726.50	739.21	25.42	X2(4)=9.49

Note:* Beyond the Significance Level

3. Johansen Cointegration Test

Given that the data exhibit unit roots with intercept and trend terms, and using a lag length of VAR(1), the results of the Johansen cointegration test are shown in Table 10, which identifies three significant cointegration vectors. The second column, Eigenvalue, lists the characteristic roots: 0.471146, 0.350327, and 0.275719. The third column presents the Likelihood Ratio trace test statistics: 194.5283, 112.3498, and 56.71397. The fourth and fifth columns show the 5% significance level critical values and p-values, respectively.

Table 10: Trace Test

Unrestricted Cointegration Rank Test (Trace)					
Hypothesized No. of E(s)	**Eigenvalue**	**Trace Statistic**	**5% Critical Value**	**Prob.**	**decision making**
None * $H_0: \gamma = 0\,; H_1: \gamma = 4$	0.471146	194.5283	76.97277	0.0000	reject H_0
At most 1 * $H_0: \gamma = 1\,;\ H_1: \gamma = 4$	0.350327	112.3498	54.07904	0.0000	reject H_0
At most 2 * $H_0: \gamma = 2\,;\ H_1: \gamma = 4$	0.275719	56.71397	35.19275	0.0001	reject H_0
At most 3 $H_0: \gamma = 3\,;\ H_1: \gamma = 4$	0.094061	15.10169	20.26184	0.2206	acceptH_0
At most 4 $H_0: \gamma = 4\,;\ H_1: \gamma = 4$	0.018118	2.358696	9.164546	0.7058	acceptH_0

Trace test indicates 3 cointegrating eqn(s) at the 0.05 level . * denotes rejection of the hypothesis at the 0.05 level

The first column, Hypothesized No. of CE(s), tests the null hypothesis H_0 of no cointegration. According to Table10 , NONE, At most 1, At most 2, and At most 3, where P-VALUE < 0.05, reject the null hypothesis, indicating the presence of cointegration vectors. For At most 3 and At most 4, where P-VALUE > 0.05, the null hypothesis is not rejected, indicating no significant cointegration vectors. Further, Table 11 shows the results of the Maximum Eigenvalue test, confirming the results of the trace test. Therefore, this study identifies three cointegration vectors among the variables. The cointegration trend graphs for each vector are shown in Figure 3.

Table 11: Maximum Eigenvalue Test

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)					
Hypothesized No. of E(s)	**Eigenvalue**	**MAX-Eigen Statistic**	**5% Critical Value**	**Prob.**	**decision making**
None * $H_0: \gamma = 0$; $H_1: \gamma > 0$	0.471146	82.17850	34.80587	0.0000	reject H_0
At most 1 * $H_0: \gamma = 1$; $H_1: \gamma > 1$	0.350327	55.63584	28.58808	0.0000	reject H_0
At most 2 * $H_0: \gamma = 2$; $H_1: \gamma > 2$	0.275719	41.61228	22.29962	0.0000	reject H_0
At most 3 $H_0: \gamma = 3$; $H_1: \gamma > 3$	0.094061	12.74299	15.89210	0.1469	acceptH_0
At most 4 $H_0: \gamma = 4$; $H_1: \gamma > 4$	0.018118	2.358696	9.164546	0.7058	acceptH_0

Trace test indicates 3 cointegrating eqn(s) at the 0.05 level . * denotes rejection of the hypothesis at the 0.05 level

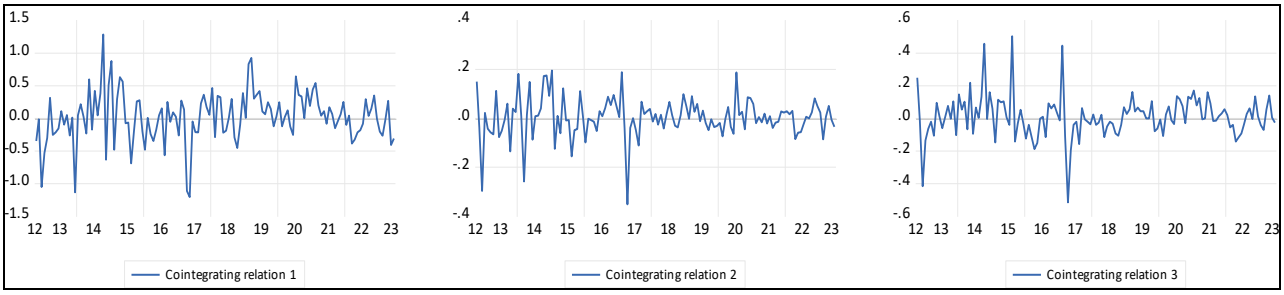


Fig 3: Trend of Cointegration for Each Group

Tables 10 and 11 indicate that with γ = 3, the null hypothesis H0H_0H0 is accepted, suggesting that there are three long-term stable cointegration relationships among the five variables: pre-sale homes, old homes, new homes, new second-hand homes, and second-hand homes. This implies that changes in the price of pre-sale homes have long-term relationships with the other variables. Short-term price fluctuations are analyzed using the Vector Error Correction Model (VECM), with lag length p=1 as the appropriate model. The linear cointegration relationships for the variables pre-sale homes ($y_{1,t}$), new second-hand homes ($y_{2,t}$), second-hand homes ($y_{3,t}$), new homes ($y_{4,t}$), and old homes ($y_{5,t}$) with the intercept term are represented by $\hat{E}c_{1,t}$、$\hat{E}c_{2,t}$、$\hat{E}c_{3,t}$, respectively. The symbols [ ] represent the statistics. Note:*exceeding 2 standard deviations

$$y_{1,t} = -3.898y_{4,t}[-8.167]^* + 2.905y_{5,t}[6.456]^{*}++0.179[0.195]\ldots. \quad \text{Eq5}$$

$$\hat{E}c_{1,t} = y_{1,t} + 3.898y_{4,t} - 2.905y_{5,t} - 0.179 \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{Eq6}$$

$$y_{2,t} = -1.106y_{4,t}[-14.262]^* + 0.007y_{5,t}[0.098] + 0.341[2.279]^* \quad \text{Eq7}$$

$$\hat{E}c_{2,t} = y_{2,t} + 1.106y_{4,t} - 0.007y_{5,t} - 0.341\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{Eq8}$$

$$y_{3,t} = -1.488y_{4,t}[-11.377]^* + 0.305y_{5,t}[2.477]^* + 0.945[3.748]^*\ldots \quad \text{Eq9}$$

$$\hat{E}c_{3,t} = y_{3,t} + 1.488y_{4,t} - 0.305y_{5,t} - 0.945 \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{Eq10}$$

Note:*exceeding 2 standard deviations

First Cointegration Equation: The first cointegration equation, Eq5, represents the long-term trend of pre-sale homes. It shows that:1.The impact of pre-sale homes ($y_{1,t}$) on new homes ($y_{4,t}$) is statistically significant with a t-statistic of [-8.167], indicating a significant negative effect.2.The impact of pre-sale homes ($y_{1,t}$)on old homes ($y_{5,t}$) is statistically significant with a t-statistic of [6.456], indicating a significant positive effect.Second Cointegration Equation: The second cointegration equation, Eq7, represents the long-term trend of new second-hand homes. It shows that:1.The impact of new second-hand homes ($y_{2,t}$) on new homes ($y_{4,t}$) is statistically significant with a t-statistic of [-14.262], indicating a significant negative effect.2.The impact of new second-hand homes ($y_{2,t}$) on old homes ($y_{5,t}$) has a t-statistic of [0.098], which is not statistically significant, indicating no significant positive effect.Third Cointegration Equation: The third cointegration equation, Eq9, represents the long-term trend of second-hand homes. It shows that:1.he impact of second-hand homes ($y_{3,t}$) on new homes ($y_{4,t}$) is statistically significant with a t-statistic of [-11.377], indicating a significant negative effect.2.The impact of second-hand homes ($y_{3,t}$) on old homes ($y_{5,t}$) is statistically significant with a t-statistic of [2.477], indicating a significant positive effect.

Vector Error Correction Model (VECM(1)) ExampleUsing the first cointegration equation as an example for the Vector Error Correction Model (VECM(1)):

$$\hat{\Delta}y_{1,t} = -0.097\hat{E}c_{1,t-1} - 0.238\Delta y_{1,t-1} + 0.155\Delta y_{2,t-1} - 0.184\Delta y_{3,t-1} + 0.211\Delta y_{4,t-1} + 0.011\Delta y_{5,t-1}\ldots\ldots \quad \text{Eq11}$$

$$\hat{\Delta}y_{2,t} = 0.049\hat{E}c_{1,t-1} + 0.062\Delta y_{1,t-1} - 0.066\Delta y_{2,t-1} - 0.026y\Delta_{3,t-1} - 0.207\Delta y_{4,t-1} - 0.119\Delta y_{5,t-1}\ldots\ldots \quad \text{Eq12}$$

$$\hat{\Delta}y_{3,t} = 0.129\hat{E}c_{1,t-1} - 0.108\Delta y_{1,t-1} - 0.073\Delta y_{2,t-1} - 0.161\Delta y_{3,t-1} - 0.083\Delta y_{4,t-1} - 0.102\Delta y_{5,t-1}\ldots\ldots \quad \text{Eq13}$$

$$\hat{\Delta}y_{4,t} = -0.011\hat{E}c_{1,t-1} + 0.059\Delta y_{1,t-1} - 0.264\Delta y_{2,t-1} - 0.166\Delta y_{3,t-1} - 0.081\Delta y_{4,t-1} + 0.023\Delta y_{5,t-1}\ldots\ldots\ldots \quad \text{Eq14}$$

$$\hat{\Delta}y_{5,t} = -0.0249\hat{E}c_{1,t-1} + 0.252\Delta y_{1,t-1} - 0.419\Delta y_{2,t-1} - 0.025\Delta y_{3,t-1} - 0.228\Delta y_{4,t-1} - 0.095\Delta y_{5,t-1}\ldots\ldots \quad \text{Eq11}$$

In the pre-sale homes equation$\hat{\Delta}y_{1,t}$ the coefficient of $\hat{E}c_{1,t-1}$ is negative, which indicates that when the long-term price deviates from the short-term price in the long-term equilibrium relationship (cointegration relationship), for example, when the long-term pre-sale home price equation Eq11 shows $y_{1,t} > 3.898y_{4,t} + 2.905y_{5,t}$+0.179, the coefficients of $-0.097\hat{E}c_{1,t-1}$ in the short-term pre-sale home price $-0.011\hat{E}c_{1,t-1}$ in new homes, $-0.0249\hat{E}c_{1,t-1}$ in old homes are negative. This implies that the prices will fall ($\hat{\Delta}y_{1,t} < 0$、$\hat{\Delta}y_{4,t} < 0$、$\hat{\Delta}y_{5,t} < 0$)) to restore the long-term equilibrium.

Similarly, in the new second-hand homes equation Eq12, the coefficient $0.049\hat{E}c_{1,t-1}$ for new second-hand homes ($\hat{\Delta}y_{2,t}$) and the coefficient$0.129\hat{E}c_{1,t-1}$ for second-hand home are positive. Thus, when $y_{1,t} > 3.898y_{4,t} + 2.905y_{5,t}$+0.179 in the cointegration relationship of pre-sale homes, the prices of new homes and new second-hand homes will rise ($\hat{\Delta}y_{2,t} > 0$、$\hat{\Delta}y_{3,,t} > 0$) to restore the long-term equilibrium.。

4. Vector error correction causality analysis and VECM Impulse Response

In the Vector Error Correction Model (VECM), understanding the short-term interactions among housing cycle variables and using Granger causality tests to check the causal relationships between variables helps to determine the "leading" or "lagging" relationships among different housing cycles. Impulse response analysis allows us to observe how variables dynamically adjust over time in response to external shocks, with a focus on changes caused by a one-standard-deviation shock to an endogenous variable.

The Impulse Response Function (IRF) describes how variables in the system adjust dynamically over time following an external shock. It shows:1.Immediate Response: Changes in consumption in the period when the shock occurs.2.Short-term Response: Changes in consumption in the subsequent few periods.3.Long-term Response: The process of consumption returning to equilibrium in the long term. The IRF plots for shocks are shown in Figures 4, 5, 6, and 7.

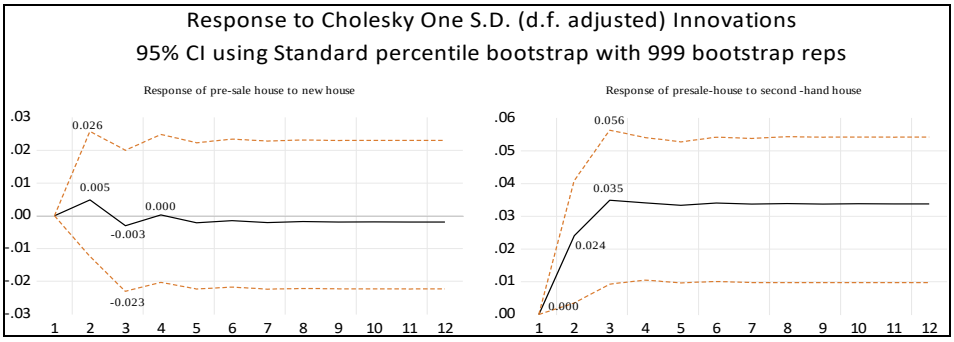


Fig 4: Impact of new and used house variables on pre-sale house trends

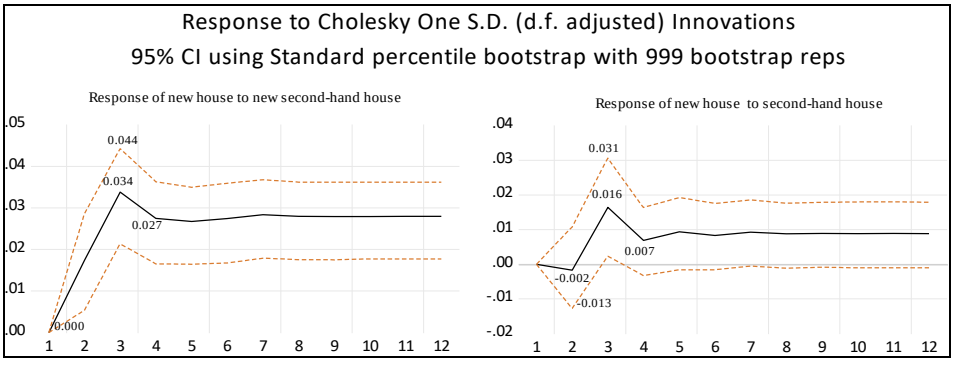


Fig 5: Impact of new and used house variables on new house trends

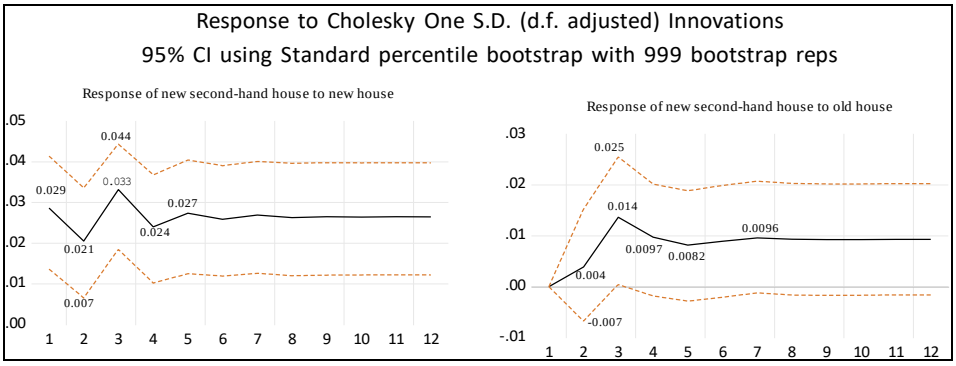


Fig 6: Impact of new and old house variables on trends in new and used house

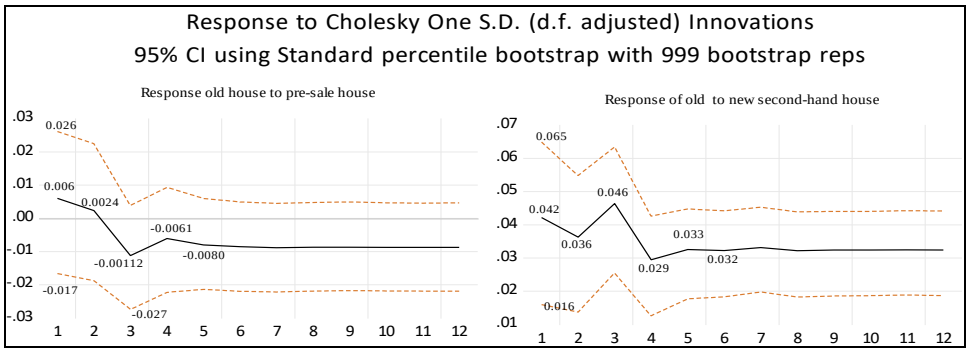


Fig 7: Impact of Pre-Sale and New Used house Variables on Old house Variables

The horizontal axis represents time periods (e.g., period 0 indicates the current period, period 1 indicates the next period), while the vertical axis shows the response values of variables (e.g., percentage changes or absolute values). The shock line represents the response values of the shocked variable at each period, with confidence intervals usually shown by dashed lines, indicating the range of uncertainty in the response values.The IRF results in this study indicate the maximum shock magnitudes as follows,show in Table12,13,14,15 and 16,For Explame,1.The largest shock is the impact of second-hand homes on pre-sale homes (0.035).2.The next largest is the impact of new second-hand homes on new homes (0.034).3.The smallest shock is the impact of pre-sale homes on old homes (0.006).4.The next smallest is the impact of new second-hand homes on old homes (0.007).

Table 12: Dependent variable: Pre-Sale Housing Variables ($y_{1,t}$)

Excluded	Chi-sq	df	Prob.
New housing ($y_{4,t}$)	3.554011	1	0.0594*
New-second housing ($y_{2,t}$)	1.466325	1	0.2259
Second housing ($y_{3,t}$)	4.938019	1	0.0263*
Old housing ($y_{5,t}$)	0.027347	1	0.8687
All	9.102335	4	0.0586

Note: $\mathcal{P}^*$ <0.10

Table 13: Dependent variable New Housing Variables ($y_{4,t}$)

Excluded	Chi-sq	df	Prob.
re-sale housing ($y_{1,t}$)	0.674026	1	0.4117
New-second housing ($y_{2,t}$)	5.692588	1	0.0170*
second housing ($y_{3,t}$)	5.406099	1	0.0201*
Old housing ($y_{5,t}$)	0.154453	1	0.6943
All	19.70980	4	0.0006*

Note: $\mathcal{P}*$ <0.10

Table 14: Dependent variable: New-second housing Variables ($y_{2,t}$)

Excluded	Chi-sq	df	Prob.
Pre-sale housing ($y_{1,t}$)	0.804530	1	0.3697
New housing ($y_{4,t}$)	4.854859	1	0.0276*
Second housing ($y_{3,t}$)	0.144701	1	0.7037

Old housing ($y_{5,t}$)	4.440146	1	0.0351*
All	14.56593	4	0.0057*
Note: $\mathcal{P}$* <0.10			

Table 15: Dependent variable: second housing variable ($y_{3,t}$)

Excluded	Chi-sq	df	Prob.
Pre-sale housing ($y_{1,t}$)	0.948758	1	0.3300
New housing ($y_{4,t}$)	0.310400	1	0.5774
New-second housing ($y_{2,t}$)	0.182267	1	0.6694
Old housing ($y_{5,t}$)	1.310942	1	0.2522
All	4.584751	4	0.3326
Note: $\mathcal{P}$* <0.10			

Table 16: Dependent variable: old housing variable ($y_{5,t}$)

Excluded	Chi-sq	df	Prob.
Pre-sale housing ($y_{1,t}$)	4.298713	1	0.0381*
New housing ($y_{4,t}$)	1.936845	1	0.1640
New-second housing ($y_{2,t}$)	4.971561	1	0.0258*
Second housing ($y_{3,t}$)	0.043003	1	0.8357
All	17.14729	4	0.0018*
Note: $\mathcal{P}$* <0.10			

5.Variance Decomposition:

Helps us understand the dynamic relationships between different variables in the system by showing the proportion of the forecast error of each variable that can be explained by itself or other variables. For this study, we used monthly data and observed over 12 periods, comparing the explanatory power of each variable. The highest self-explanatory power was found as follows Table 17 :1. Pre-sale homes: 73.77%.2.New homes: 66.74%.3.New second-hand homes: 61.83%.4.Second-hand homes: 43.44%.5.Old homes: 47.37%.Second-hand and old homes are significantly influenced by other variables.

Table 17: Average Percentage of Variance Decomposition

Exogenous Variables/ Endogenous Variables	Pre-sale housing	New housing	New-econd housing	Second housing	Old housing
Pre-sale housing($y_{1,t}$)	73.77	0.11	4.15	20.91	1.06
New housing ($y_{4,t}$)	2.23	66.74	26.18	3.29	1.56
New-second housing ($y_{2,t}$)	3.51	29.46	61.83	2.40	2.79
Second housing ($y_{3,t}$)	4.37	22.04	28.60	43.33	1.66
Old housing ($y_{5,t}$)	1.25	20.75	29.09	1.57	47.34

6. Results

Hypothesis 1. Verify whether there is a co-integration relationship between the long-term price trends of each stage of the housing life cycle. There are three sets of co-integrated vector equations among the variables in this study. For example, the first set is that the correction of short-term house prices for pre-sale houses, new houses, and old houses is in the opposite direction to the long-term equilibrium house price; Correction for the same phase

Hypothesis 2. Verify the short-term error correction of price fluctuations in each stage of the housing life cycle, and whether the variables have a leading or lagging relationship. The leading-lag relationship in each stage of the housing life cycle is that new houses lead pre-sold houses, pre-sold houses lead old houses, old houses lead new noon houses, and new mid-old houses lead new houses, that is, pre-sold houses → old houses → new Second-hand houses → new houses → pre-sold houses form a one-way cyclic relationship. As for second-hand houses, they are the leading pre-sold houses and new houses.

Hypothesis 3. Whether the self-explanation ability of the variables reaches 50% explanatory power. The self-explanation level of pre-sold houses is 73.77%, that of new houses is 66.74%, that of new and second-hand houses is 61.83%, that of second-hand houses is 43.44%, and that of old houses is 47.37%. The explanatory power of the impact of medieval houses and old houses did not reach 50%, which is relatively low compared with other variables.

7. Conclusion

This study found that the fluctuation of housing prices in Taichung City has a long-term equilibrium co-integration relationship between housing prices. The error correction of pre-sold houses, new houses and old houses is an inverse complementary relationship. The error correction efficiency of pre-sold houses is higher than that of new houses and old houses. ; The error correction between the new ancient house and the ancient house is a substitution relationship of correction in the same direction, and the error correction efficiency of the ancient house is higher than that of the new ancient house. The research results of Hua (2001) pointed out that the current price of pre-sale prices (prices of existing houses) will tend to be long-term equilibrium, and the pre-sale prices adjust faster than the existing prices (prices of existing houses). This study takes a more in-depth look at the long-term equilibrium relationship and trend of existing

housing prices at each stage.

The short-term error correction verification results found that the relationship between the leading and lagging of housing price error correction at each stage is that new houses lead pre-sold houses, pre-sold houses lead old houses, old houses lead new noon houses, and new middle-aged houses lead new houses, that is, pre-sold houses → Old houses → new second-hand houses → new houses → pre-sold houses form a one-way cyclic relationship. As for the second-hand houses, they are the leading pre-sold houses and new houses. Research by Hong Shujuan (2008) pointed out that pre-sold houses lead the price of old houses and their research on pre-sold houses Prices lead the stock market, and the stock market leads second-hand houses, so pre-sale houses lead second-hand houses. The second-hand houses in this study generally refer to all existing houses. This study supplements the short-term relationship of existing houses and provides a better understanding of the leading and lagging of existing houses at each stage. This study refers to mid-century houses (more than 15 years to less than 30 years old) and old houses (more than 30 years old) as having less than 50% self-explanatory ability.

Due to its commodity nature, the price of pre-sale houses is lagging behind in information. Research literature often uses a wide range of existing house prices as the benchmark for pre-sale house pricing. When the economy is booming, fluctuations in housing prices increase, and it seems that the increase in pre-sale houses is large. Being ahead of the price of existing houses makes the price of existing houses lose the meaning of pricing benchmark. It has been proved that the leading reaction of new house prices drives the price of pre-sold houses, making the price fluctuations of existing houses at various stages greater

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